Workshop Bruhat-Tits Theory

Darmstadt, 09.09.2026 – 11.09.2026

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About the Program:
The goal of this workshop is to study Bruhat-Tits Theory over discretely valued Henselian fields.
We will mostly follow the first two parts of [KP23].

Acknowledgments:
We would like to thank Patrick Bieker for several enlightening discussions about Bruhat-Tits theory and providing further recommendations for this program. We would also like to extend our thanks to Rizacan Ciloglu, Thibaud van den Hove, and Torsten Wedhorn for additional input.

General setup:
Let k be a field given with a discrete valuation $\omega\colon k \to \mathbb{Z}\cup\{\infty\}$. Let $\mathfrak{o} = \{x \in k \mid \omega(x) \geq 0\}$ be the ring of integers, $\mathfrak{m} = \{x \in k \mid \omega(x) > 0\}$ the maximal ideal of $\mathfrak{o}$ and $\mathfrak{f} = \mathfrak{o}/\mathfrak{m}$ the residue field. We assume that $\mathfrak{o}$ is Henselian and the residue field $\mathfrak{f}$ is algebraically closed. Let G be a connected reductive group over k. Note that G is automatically quasi-split under the given assumptions.

*All of the talks marked with a * are currently planned as 90 minute-talks. If more speakers are interested in participating, we would be open to splitting those talks into two 60 minute-talks.*

Day 1 — 09.09.2026, 9:30am – 4pm, Room S2|15 315

9:30am *Registration*

10am Talk 1*: Reductive groups, root data, root groups, and affine root systems **(Speaker: ...)**

Start by reviewing reductive groups and root data. Illustrate these concepts using the examples of SL_2, SU_3, and Sp_4. Explain the classification using Dynkin diagrams and introduce root groups [KP23, Section 2.6].

Then introduce affine root systems [KP23, Section 1.3], including the affine Weyl group. Go on to explain [KP23, Construction 1.3.27] and [KP23, Theorem 1.3.63]. In particular, expand on [KP23, Figure 1.3.1].

11:30pm *Lunch break*

1:30pm Talk 2: Abstract buildings and Tits systems **(Speaker: ...)**

After introducing the concepts of (poly-)simplicial complexes and chamber complexes [KP23, Definition 1.5.1], define the notion of an abstract building [KP23,

Definition 1.5.5]. Also explain the order relation (induced by inclusion) on (poly-)simplicial complexes [KP23, Remark 1.5.4].
As the second part of your talk, explain the notion of a Tits system and its Weyl group [KP23, Definition 1.4.1 and Remark 1.4.2]. Briefly introduce parabolic subgroups [KP23, Definition 1.4.4].
As a third part of your talk, explain the connections between (poly-)simplicial complexes and Tits systems [KP23, Proposition 1.5.6]. If time permits, explain [KP23, Proposition 1.5.13].

2:30pm *Coffee & Cake*

3pm Talk 3: Existence of the building and construction for SL_2 and SU_3 **(Speaker: ...)**

Explain [KP23, Axiom 4.1.1] on the "Existence of a building". Further introduce the notions of parahoric and Iwahori subgroups [KP23, Definition 4.1.3].
In the second part of your talk, follow [KP23, Chapter 3] to explain the contruction of the building for the cases of SL_2 and SU_3. In particular, describe the structure of parahoric subgroups.

Day 2 — 10.09.2026, 9:30am – 3:30pm, Room S2|15 315

9:30am Talk 4: Axiom on the affine root system **(Speaker: ...)**

Introduce [KP23, Axiom 4.1.6] on the "Affine root system" and explain the affine space as introduced in [KP23, Axiom 4.1.4]. Use the concrete construction of the Bruhat-Tits building for SL_2 and SU_3 from Talk 3 (see also [KP23, Chapter 3]) to illustrate these axioms and the concept of affine root systems as much as possible.

10:30am *Small break*

11am Talk 5: Decompositions **(Speaker: ...)**

State the Iwahori decomposition [KP23, Axiom 4.1.16]. Briefly introduce [KP23, Fact 4.1.25] and present the Cartan decomposition in [KP23, Theorem 5.2.1] with the accompanying proof. Explicitly write out the Cartan decomposition in the case of GL_n.[1]

Proceed similarly with the Iwasawa decomposition [KP23, Theorem 5.3.3] and state [KP23, Theorem 5.3.4].

12pm *Lunch break*

2pm Talk 6*: Construction of the apartment, affine reflections, and roots **(Speaker: ...)**

The main goal of this talk is to showcase the construction of the Bruhat-Tits apartment in the case of quasi-split reductive groups. Start by describing the construction in [KP23, Section 6.1.(a)]. Accompany this discussion with the example of SL_2 in [KP23, Section 3.1], and take particular notice of the groups $U_{A,\varphi,r}$. Then show the extensions to quasi-split reductive groups [KP23, Sections 6.1.(b) - (c)] using the example of SU_3 as in [KP23, Section 3.2]. Touch on Sp_4 and SL_3 as their buildings have dimension 2 and connect this with [KP23, Figure 1.3.1].

Continue with the description of affine roots and the simplicial structure of $\mathcal{A}$ following [KP23, Section 6.3] focusing on the quasi-split case wherever it is convenient to do so. Lastly, if time permits, discuss the explicit descriptions in [KP23, Section 6.4(a) and (b)].

evening *Dinner*

[1] See https://personal.math.ubc.ca/~cass/research/pdf/GLN.pdf, Bill Casselman.

Day 3 — 11.09.2026, 9:30am – 4pm, Room S2|15 315

9:30am Talk 7: Bruhat-Tits building **(Speaker: ...)**

State [KP23, Axiom 4.1.9] as the goal of the talk. Continue by showing an alternative definition of parahoric subgroups following [KP23, Definition 2.6.23, 7.4.1, and 7.4.5]. Include the general discussions from [KP23, Section 7.3] as necessary. Then sketch the proof of [KP23, Theorem 7.5.1] providing as many details as time permits. Conclude the talk by stating [KP23, Proposition 7.6.4] showing that the alternative definitions coincide with those of [KP23, Definition 4.1.3] and Talk 3.

10:30am *Small break*

11am Talk 8*: Iwahori-Weyl group **(Speaker: ...)**

Start by giving a quick overview over the topics of Neron models and Coxeter groups as far as necessary for the rest of your talk.

Recall the definition of the affine Weyl group [KP23, Section 1.3] and the extended affine Weyl group [KP23, Definition 1.3.71]. Next, explain the definition of the Iwahori-Weyl group and relative vector Weyl group using tori as in [HR08, Definition 7] or [Ric13, Definition 1.1]. Explain how to define the structure of a "quasi-Coxeter group" on the Iwahori-Weyl group. This can be found in [HR08, Lemma 14] or [Ric13, Remark 1.4].

State and if time permits, prove [KP23, Proposition 6.6.2 and 6.6.3].

The following sources might be helpful: [KP23, Section B.7, 8.1, 8.2, 7.8, and 6.6], [AR25, Section 4.1], [Ric13, Section 1], and [HR08].

12:30pm *Lunch break*

2:30pm Talk 9: Construction of parahoric integral models **(Speaker: ...)**

Start by explaining [KP23, Axiom 4.1.20 and Definition 4.1.21] and illustrate them using SL_2 [KP23, Example 8.3.5]. Continue by presenting the core ideas of the general construction. In particular, touch on the proofs of [KP23, Proposition 8.3.1, Theorem 8.3.2, and Theorem 8.3.13]. As time permits, feel free to include additional background details from [KP23, Section 8.1 and 8.2].

The speaker should feel free to include further results. For instance, [KP23, Section 8.4] might be of general interest.

3:30pm *Coffee, Cake & Final Discussion*

References

[AR25] P. N. Achar and S. Riche. *Central Sheaves on Affine Flag Varieties*. 2025.

[HR08] T. Haines and M. Rapoport. "Appendix: On parahoric subgroups". In: *Advances in Mathematics* 219.1 (2008), pp. 188–198. ISSN: 0001-8708.

[KP23] T. Kaletha and G. Prasad. *Bruhat–Tits theory: a new approach*. Vol. 44. Cambridge University Press, 2023.

[Ric13] T. Richarz. "Schubert varieties in twisted affine flag varieties and local models". In: *Journal of Algebra* 375 (2013), pp. 121–147. ISSN: 0021-8693.