GAUS AG ON THE MOD p RIEMANN–HILBERT CORRESPONDENCE

1. Time and place

Mondays, 10:00 to 12:00, in the Hilbert room

2. References

The main reference is

- *The mod p Riemann–Hilbert correspondence and the perfect site* by Akhil Mathew, available at `https://arxiv.org/pdf/2205.11657`.

Other useful references are:

- *A Riemann–Hilbert correspondence in positive characteristic* by Bhargav Bhatt and Jacob Lurie, available at `https://arxiv.org/pdf/1711.04148`
- *The Riemann–Hilbert correspondence for unit F-crystals* by Mathew Emerton and Mark Kisin, available at `https://math.uchicago.edu/~emerton/pdffiles/RH.pdf`
- *Lectures on Condensed Mathematics* by Peter Scholze, available at `https://www.math.uni-bonn.de/people/scholze/Condensed.pdf`

3. Program

If not mentioned otherwise, the numbering refers to the paper by Akhil Mathew.

27.10.	Introduction	Tom	
3.11.	Étale cohomology	Lorenzo	Recall and/or prove enough about étale cohomology theory to prove that affine $\mathbb{F}_p$-schemes have étale cohomological dimension ≤ 1 (see Example 2.2).
10.11	Prestable and derived categories	Timon	§2 up to Remark 2.5: Explain the category $\mathcal{D}(\mathcal{T})_{p-\mathrm{tors}}$. Prove Proposition 2.1 and apply it with the small étale site and with the perfect site (Examples 2.2 and 2.3). Explain the adjunction $\pi^* \dashv \pi_*$ and prove its properties (Proposition 2.4 and Remark 2.5). Review some of the theory of prestable categories as needed.
17.11.	Frobenius modules	?	§3 up to Proposition 3.10: Define the ring $W(R)[F^{\pm 1}]$ (Construction 3.1). Define algebraic $W(R)[F^{\pm 1}]$-modules (Definition 3.3) and prove some properties of the category $\mathcal{D}_{\mathrm{alg}}(W(R)[F^{\pm 1}])$ (Corollary 3.5 and Proposition 3.7). Explain the extension and restriction of scalars functors (Remark 3.2) and prove Propositions 3.8 and 3.9. Prove Proposition 3.10.
24.11.	The Breen–Deligne resolution (1)	Georg	This talk and the next one cover Scholze's lectures on condensed mathematics, appendix to lecture IV (the numbering refers to these notes): Define Breen–Deligne resolutions for abelian group objects in $\mathrm{Sh}(\mathcal{T})$ for any site $\mathcal{T}$. Prove the existence of functorial Breen–Deligne resolutions for abelian groups (Theorem 4.10). Deduce the corresponding statement for abelian group objects in a sheaf topos (Remark 4.7).
1.12.	The Breen–Deligne resolution (2)	Georg	
8.12.	Breen's theorem (1)	Luca	§A.1: State Breen's theorem (Theorem A.1). Explain ind-affine schemes as in Definition A.2. Explain rational Witt vectors as in Construction A.4 and big Witt vectors as in Constrution A.5. State and prove (as much as possible of) Proposition A.6.
15.12	Breen's theorem (2)	Luca	§A.2: Prove Theorem A.1 and Corollary A.16.
5.1.	Frobenius modules and sheaves	Julie	§4: Construct the functor $W \otimes^{\mathbb{L}}_{W(R)[F^{\pm 1}]} (-)$ as in Construction 4.1. Explain Remark 4.2 and prove Theorem 4.3.

12.1.	The (covariant) RH correspondence	Timo	§5, up to the proof of Theorem 1.3: Define the covariant Riemann–Hilbert functor $\mathrm{RH}_{\mathrm{cov}}$ (Construction 5.3). State and prove Theorems 5.5 and 1.3. If time permits, explain compatibilities of $\mathrm{RH}_{\mathrm{cov}}$ (Remark 5.10).
19.1.	Compact objects	Klaus	Rest of §2 and §3, culminating in Remark 5.9: Define locally regular coherent categories (Definition 2.6). Prove the characterization of compact objects of $\mathcal{D}(X_{\mathrm{et}})_{p-\mathrm{tors}}$ (Proposition 2.8). Define holonomic $W(R)[F^{\pm 1}]$)-modules (Definition 3.11) and prove as much as possible of Proposition 3.15. Explain Remark 5.9.
26.1.	Unit Frobenius modules	Anton	§6.1: Define unit $R[F]$-modules (Definition 6.1) and explain Remark 6.2. Explain unitalization (Construction 6.4) and prove its universal property (Proposition 6.5). Prove Proposition 6.7 and Corollary 6.9. Define finitely generated unit $R[F]$-modules (Definition 6.10) and explain Proposition 6.11 and Corollary 6.12.
2.2.	The EK correspondence (1)	Manuel	This talk and the next one cover §6.2: Define the solutions functor Sol (Construction 6.14). Prove that the restriction of Sol to unit $R[F]$-modules of bounded projective amplitude is fully faithful (Theorem 6.16). State and prove (as much as possible of) the contravariant Riemann–Hilbert correspondence (Theorem 6.20).
9.2.	The EK correspondence (2)	Manuel	