Connecting Young Women in Geometry and Arithmetic

Heidelberg, May 26-28, 2025

Schedule

Monday, 26.05.2025

- 09:00 - 10:00 Registration
- 10:00 - 11:00 Josefien Kuijper (Utrecht)
- 11:30 - 12:30 Catrin Mair (Darmstadt)
- 12:30 - 14:00 Lunch
- 14:00 - 15:00 Dzoara Selene Nuñez-Ramos (Wuppertal)
- 15:30 - 16:30 Vivien Picard (Wuppertal)

Tuesday, 27.05.2025

- 09:00 - 10:00 Clémentine Lemarié--Rieusset (Essen)
- 10:30 - 11:30 Louisa Bröring (Essen)
- 11:45 - 12:45 Anneloes Viergever (Hanover)
- 12:45 - 14:15 Lunch
- Free afternoon with organized hike
- 19:30 Dinner

Wednesday, 28.05.2025

- 09:30 - 10:30: Katharina Müller (Unibw Munich)
- 11:00 - 12:00: Xenia Dimitrakopoulou (Aix-Marseille)
- 12:00 - 13:30 Lunch
- 13:30 - 14:30: Hind Souly (Essen)
- 15:00 - 16:00: Siqi Yang (LSGNT London)

Titles and Abstracts

Louisa Bröring *The $\mathbb{A}^1$-Euler Characteristic of Symmetric Powers*

The $\mathbb{A}^1$-Euler characteristic of a smooth, projective scheme over a field of characteristic not two is an algebro-geometric analogue of the topological Euler characteristic. More precisely, it is a motivic measure valued not in the integers, but rather in (virtual) quadratic forms, and it carries a lot of information. For example, for a smooth, projective scheme X over $\mathbb{R}$, the rank of the $\mathbb{A}^1$-Euler characteristic is the topological Euler characteristic of $X(\mathbb{C})$ and its signature is the topological Euler characteristic of $X(\mathbb{R})$. In general, $\mathbb{A}^1$-Euler characteristics are hard to compute; for example, not much is known about its behaviour under taking quotients.

In this talk, we give a gentle introduction to this invariant and we provide an overview on what is known about the $\mathbb{A}^1$-Euler characteristic of the symmetric powers of a quasi-projective scheme. Furthermore, we partially confirm a conjecture of Pajwani-Pál by computing the $\mathbb{A}^1$-Euler characteristic of the symmetric powers of a split toric variety and a curve. The latter computation is joint work with Anna Viergever.

Xenia Dimitrakopoulou

Josefien Kuijper

Clémentine Lemarié--Rieusset *Motivic linking in the projective space*
In classical knot theory and in projective knot theory, one can associate to an oriented link with two components (i.e. two disjoint oriented knots / projective knots) its linking number: an integer which counts how many times one of the components turns around the other component. In this talk, I will present motivic counterparts (called quadratic linking degrees) over a perfect field F, to the linking number, which describe how two (nice) disjoint oriented closed F-subschemes of an ambient F-scheme can be intertwined, *i.e.* linked together. I will then focus on the linking of two embedded projective lines $\mathbb{P}^1$ in the projective space $\mathbb{P}^3$, in which case the quadratic linking degree couple is a couple of elements of the Witt group of F (which, in characteristic different from 2, is a set of equivalence classes of quadratic forms together with the orthogonal direct sum). After giving several examples in this context, I will end the talk with a brief discussion of other contexts.

Catrin Mair
The ∞-category Cond(Ani) of condensed anima combines the homotopy-theoretic direction of anima with the topological space direction of condensed sets. For example, we can recover the shape of a sufficiently nice topological space from

the corresponding condensed anima. My talk will focus on a joint refinement of the étale homotopy type and the pro-etale fundamental group of a scheme, realised as an object in Cond(Ani). This is closely related to work of Barwick, Glasman and Haine on exodromy. I will give an overview of results from my dissertation and an ongoing project jointly with Haine, Holzschuh, Lara and Wolf.

Katharina Müller *Iwasawa theory of graphs, Igusa towers and number fields.*

We will introdcue Isogeny graphs and will relate their Ihara L-functions to Hasse-Weil L-functions of modular curves over finite fields. We will then explain Iwasawa theory for graphs in the case of $\mathbb{Z}_p$- and $\mathrm{GL}_2(\mathbb{Z}_p)$-towers and explain relations to the corresponding Iwasawa theories of number fields and Igusa towers.

Dzozara Nuñez-Ramos *Cohomology of a certain wild quotient singularity*

Let k be an algebraically closed field. In characteristic 2, the isolated wild quotient singularities arising from an action of the cyclic group $\mathbb{Z}/2\mathbb{Z}$ on the power series ring in two variables $k[\![u, v]\!]$, were completely described by Artin as an explicit hypersurface singularity. Peskin extended Artin's result in characteristic 3. More recently, Schröer and Lorenzini introduced moderately ramified actions in any characteristic $p > 0$ on the formal power series ring $k[\![u_1, \ldots, u_n]\!]$ with $n \geq 2$, which give rise to a new class of wild quotient singularities.

Motivated by this construction, we will describe in this talk the resolution of singularities of the singularity arising from a specific moderately ramified action on $k[\![u, v]\!]$ in any characteristic and we aim to compute the cohomology of the structure sheaf of this resolution, which is an invariant of this singularity. This is part of my PhD thesis which is work in progress.

Viven Picard *Constructing Logarithmic Hodge Numbers in Dimension 2*

Let k be an algebraically closed field of positive characteristic p. For a variety X, the Hodge of X numbers are defined as

$$h^{r,q}(X) := \dim_k H^q(X, \Omega^r_X).$$

If X is a smooth and projective k-variety of dimension n, the Hodge numbers of X always satisfy the two relations

(1) $h^{0,0}(X) = 1$,

(2) $h^{r,q}(X) = h^{n-r,n-q}$.

Conversely, Remy van Dobben de Bruyn and Matthias Paulsen showed in 2020 that for any collection of integers $(a^{r,q})_{0\leq r,q\leq n}$ and $m \geq 2$ such that $a^{0,0} = 1$ and

$a^{r,q} = a^{n-r,n-q}$ for all $0 \leq r, q, \leq n$, there exists a smooth projective k-variety X of dimension n such that

$$h^{r,q}(X) \equiv a^{r,q} \mod m.$$

Similar to the Hodge numbers, one can define the logarithmic Hodge numbers of a smooth projective variety X over k,

$$h^{r,q}_{\log}(X) := \dim_{\mathbb{F}_p} H^q_{\text{ét}}(X, \Omega^r_{X,\log}),$$

where

$$\Omega^r_{X,\log} := \ker(1 - C^{-1} : \Omega^r_X \to \Omega^r_X / B^r_X),$$

with $B^r_X := d(\Omega^{r-1}_X)$ the image under the differential of the de Rham complex, are the logarithmic differential forms. The question arises as to whether a similar result as the one of Remy van Dobben de Bruyn and Matthias Paulsen can be achieved also for the logarithmic Hodge numbers. In my talk, I will focus on the case of smooth projective surfaces. I will show that for any integers a_1, a_2 and $m \geq 2$, there exists a smooth projective surface S such that

$$h^{0,1}(S) \equiv a_1 \mod m,$$
$$h^{0,2}(S) \equiv a_2 \mod m.$$

To achieve this, I will work with weakly ordinary varieties.

Hind Souly *On strong F-regularity of rings defined by commutators in* $\mathfrak{sl}_n$.

Let k be a finite field of characteristic $p > 0$, where p is an odd prime number. For $n \in \mathbb{N}$, $g \geq 12$ and $i \in \{1, \cdots, g\}$, we define X_i, Y_i to be matrices of indeterminates in the special linear Lie algebra $\mathfrak{sl}_n(k)$. In this talk, we will outline our strategy to prove strong F-regularity of the following ring $k[X_1, Y_1, \cdots, X_g, Y_g]/([X_1, Y_1] + \cdots + [X_g, Y_g])$, where $[X, Y] = XY - YX$, for all $n \in \mathbb{N}$ not divisible by p. Similar rings were studied by Aizenbud-Avni, Budur, and Simpson over the complex numbers; our goal is to establish analogous results in characteristic p.

Anneloes Viergever *Quadratic Donaldson-Thomas invariants: Stories and Computations*

(Zero-dimensional) Donaldson-Thomas-invariants "count" things like ideal sheaves of a given length which have zero-dimensional support on a smooth projective complex threefold. Maulik, Nekrasov, Okounkov and Pandharipande have proven a formula for the generating series of these Donaldson-Thomas invariants in terms of the MacMahon function in the toric case. We discuss a conjectural analogue of this result for smooth projective real threefolds satisfying an orientation condition, using a quadratic version of Donaldson-Thomas invariants taking values in Witt rings which are constructed using work of Levine. We provide evidence for the conjecture coming from computations for $\mathbb{P}^3$ and $(\mathbb{P}^1)^3$. This talk is based on my thesis and on joint work with Marc Levine.

Siqi Yang *On the geometric Serre weight conjecture for Hilbert modular forms*

Let p be a prime and $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{F}_p})$ be a continuous, odd, irreducible representation. The weight part of Serre's conjecture predicts the minimal weight $k(\geq 2)$ such that ρ arises from a modular eigenform of weight k and level prime to p. It is refined by Edixhoven to include the weight one forms by viewing mod p modular forms as sections of certain line bundles on the special fibre of a modular curve. An important generalisation of the weight part of Serre's conjecture is formulated by Buzzard, Diamond and Jarvis by considering a totally real field F and Hilbert modular forms. Later, a geometric Serre weight conjecture is formulated by Diamond and Sasaki in the spirit of Edixhoven's refinement. I will discuss the relation between the geometric Serre weight conjecture and the Buzzard–Diamond–Jarvis conjecture in the quadratic case.

Organizers

Morten Lüders (Heidelberg)
Alberto Merici (Heidelberg)
Sabrina Pauli (Darmstadt)