Berkovich motives

GAUS AG summer term 2025

We are studying Scholze's recent preprint on *Berkovich motives* where a theory of (étale) Berkovich motives is constructed. This is closely related to Ayoub's theory of rigid-analytic motives, but works uniformly in the archimedean and nonarchimedean setting. Applying the theory to discrete fields, one still recovers the étale version of Voevodsky's theory. Two notable features of Berkovich motives which do not hold in other settings are that over any base, the cancellation theorem holds true, and under only minor assumptions on the base, the stable ∞-category of motivic sheaves is rigid dualizable.

Time and Place: There will be sessions of two talks alternating between Heidelberg, Darmstadt, and Frankfurt. First talk 13:00-14:30, second talk 15:00-16:30.

Date:	24.04.	15.05.	22.05	12.06.	26.06.	10.07.
Venue:	DA	HD	DA	?	DA	HD

After the last session on 10th July, we will have a cozy walk along Philosophenweg and dinner in the old town of Heidelberg.

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Talks

Talk 0: Overview (15min) (24.04. Alberto Merici)
Brief overview over the seminar's content.

Talk 1: Motives (24.04. Christian Dahlhausen)
Explain construction of $\mathcal{DA}^{\text{eff}}_{\text{ét}}(S,\Lambda)$ for Λ a commutative ring. Show that $\mathbb{P}^1$ is a symmetric object (see e.g. [MVW06, Prop 15.7]) and define $\mathcal{DA}_{\text{ét}}(S,\Lambda)$. Construct some realizations as an example (Betti, de Rham, ℓ-adic) and the corresponding E_∞ ring spectra.

Talk 2: Cancellation Theorem (75min) (24.04. Alberto Merici)
Recall the comparison between $\mathcal{DA}^{\text{eff}}_{\text{ét}}(k,\mathbb{Q})$ and $\mathcal{DM}^{\text{eff}}(k,\mathbb{Q})$. Explain Voevodsky's proof of the cancellation theorem for $\mathcal{DM}(k,R)$ for a field k. If time permits, give a overview of the cancellation on **Rig**$\mathcal{DA}$.

Talk 3: Berkovich spaces (08.05. Katharina Hübner)
This talk covers the content of [Sch24, §3]. Explain the basics about Banach rings and introduce the Berkovich spectrum $\mathcal{M}(A)$ of a Banach ring A. Explain the geometry of $\mathcal{M}(\mathbb{Z})$ and of the nonarchimedean unit disc $\mathcal{M}(C\langle T\rangle_1)$ for an algebraically closed nonarchimedean Banach field C.

Talk 4: The arc-topology (08.05. Jon Miles)
Exaplain the arc topology in the setting of Berkovich motives [Sch24, §3]. In particular, show that locally for the arc-topology, we can assume that our Banach rings are strictily totally

disconnected [Sch24, Prop. 3.11]. Afterwards give a sketch of [Sch24, Thm. 4.1], i.e. prove that any finitary presheaf on strictly totally disconnected Banach algebras is an arc-sheaf. Finally, prove proper base change [Sch24, Thm. 4.20] and excision [Sch24, Prop. 4.25]. Explain [Sch24, 4.26].

It is useful to note that some proofs of [Sch24, §3, §4] use ideas from [Sch22].

Talk 5: Effective motives (22.05. Christian Dahlhausen)
Goal of this talk is to cover [Sch24, §5]. More precisely, define ball-invariant sheaves and show that ball-invariance can be tested on non-discrete algebraically closed Banach fields [Sch24, Cor. 5.4]. Give the explicit formula for the left adjoint of the inclusion of ball-invariant sheaves to finitary sheaves. State without prove that with torsion coefficients étale sheaves are ball-invariant.

Explain the free generators of $\mathcal{DM}^{\mathrm{eff}}_{\mathrm{mot}}$, cf. [Sch24, Prop. 5.12, 5.13] and deduce [Sch24, Thm. 5.14]. Lastly, explain the construction of $\overline{\mathbb{G}}_{\mathrm{m}}$ and Tate twist. In particular, prove [Sch24, Prop. 5.17] and if time permits, give the idea of [Sch24, Prop. 5.19]. For [Sch24, Prop. 5.17] the theorem before [Sch24, 5.16] is crucial, so at least the idea of the proof should be explained.

Talk 6: Free motivic sheaves (22.05. NN)
The goal of this talk is to cover [Sch24, §6]. Specifically [Sch24, Thm. 6.1 and 6.3]. For this, explain the proofs of [Sch24, Lem. 6.4 and 6.5]. Give also an overview of the tilting equivalence [Sch24, Prop. 6.8] and the existence of the homotopy t-structure [Sch24, Cor. 6.8]. If time permits sketch the proof of the latter.

Talk 7: The Cancellation Theorem (12.06. Can Yaylali)
Cover [Sch24, §7].

Talk 8: K-theory (12.06. Christian Dahlhausen)
Cover [Sch24, §8].

Talk 9: $\mathcal{D}_{\mathrm{mot}}(X)$ (26.06. Chirantan Chowdhury)
Cover [Sch24, §9].

Talk 10: $\mathcal{D}_{\mathrm{mot}}(C)$ (26.06. Alberto Merici)
Cover [Sch24, §10].

Talk 11: $\mathcal{D}_{\mathrm{mot}}(k)$ (10.07. Can Yaylali)
Cover [Sch24, §11].

Talk 12: Buffer (10.07.. NN)

References

[MVW06] Carlo Mazza, Vladimir Voevodsky, and Charles Weibel. *Lecture notes on motivic cohomology*, volume 2. Clay Mathematics monographs, 2006.

[Sch22] Peter Scholze. Etale cohomology of diamonds, 2022. URL: `https://arxiv.org/abs/1709.07343`, `arXiv:1709.07343`.

[Sch24] Peter Scholze. Berkovich motives, 2024. URL: `https://arxiv.org/abs/2412.03382`, `arXiv:2412.03382`.