Schedule

10:30 - 11:00	Intro: Quadratic Forms and Galois Cohomology (Nils)
11:30 - 12:00	Introduction to N-graded Proj construction and toric varieties (Felix)
12:10 - 12:40	An overview of Galois- and Étale Cohomology (Immanuel)
Lunch Break	
14:00 - 15:00	Poitou-Tate-Duality for arithmetic Schemes (Immanuel)
Coffee and Cookies	
15:20 - 16:20	Multigraded Proj of polynomial rings (Felix)
16:30 - 17:30	Sums of Squares in Function Fields (Nils)
17:30 - 18:00	Organization, next Symposium, etc.
Pizza, drinks, and board games in the institute	

Abstracts

Nils Witt: Sums of Squares in Function Fields

Abstract: For any field K, we call the smallest positive integer n, such that any sum of squares in K can be written as a sum of n squares, the Pythagoras number $p(K)$ of K, and set $p(K) = \infty$ if no such number exists.

Let $k/\mathbb{Q}$ be a number field and let F/k be a finitely generated field extension such that k is algebraically closed in F, say $\operatorname{tr.deg}_k F = d$. Then we show that

$$p(F) \leq \begin{cases} 2^{d+1}, & d \geq 2 \\ 7, & d = 1. \end{cases}$$

Felix Göbler: Multigraded Proj of Polynomial Rings

Abstract: It is a long known result that $\mathbb{Z}$-graded rings S (of type N) give rise to projective spaces. In particular, the irrelevant ideal, usually denoted by S_+, is fixing the projective structure of $\operatorname{Proj}^N(S)$. For D-graded rings S however (where D is a finitely generated abelian group), we will see that we have many choices of a projective structure, corresponding to choices of irrelevant ideals of S. Most importantly, there will be a unique maximal choice of an irrelevant ideal by Brenner-Schröer, corresponding to the choice of all relevant elements on the ring side or to the choice of all maximal cones on the geometric side.

If S is a polynomial ring, the latter gives rise to a system of fans and allows us to define $\operatorname{Proj}^D(S)$ in terms of toric prevarieties and vice versa. In particular, this point of view will give us a separability criterion for free. Following the classical construction very closely, we want to examine which parts of it generalize and which do not.

Immanuel Klevesath: Poitou-Tate-Duality for Arithmetic Schemes

Abstract: Classical Poitou-Tate-Duality plays a central role in number theory, namely in the Galois cohomology of number fields. It provides answers to the question of when certain local-global principles hold. Thomas Geisser and Alexander Schmidt generalize this by extending the duality to arithmetic Schemes.