Conference "Anabelian and Motivic Homotopy Theory"
September 23-28, 2024, Heidelberg

Titles and Abstracts

Piotr Achinger: *Tame fundamental groups II. Log schemes beyond fs.*

This is the second of two talks on a joint project of P. Achinger, K. Hübner, M. Lara, and J. Stix. The first one was/will be given by K. Hübner.
In order to finish the proof of finite generation of the tame fundamental group of a rigid space, we solve an analogous problem concerning log schemes over the residue field and their (tame) Kummer etale covers. These log schemes arise as special fibers of semistable formal schemes over the ring of integers. Since the ring is non-noetherian (as its fraction field is algebraically closed), the log structures in question will not be locally defined by finitely generated saturated (fs) monoids. This causes serious difficulties, and in fact requires us to rebuild the foundations of logarithmic geometry beyond finitely generated monoids. Once one erases the finiteness assumption, numerous pitfalls are to be avoided, in particular related to the existence of local charts. One surprising outcome is that with the correct definitions, the maps of schemes underlying a smooth morphism of saturated log schemes might not be of finite type.

Tom Bachmann: *Real points of motivic Eilenberg–MacLane spaces*

The motivic Eilenberg–Mac Lane spaces $K(Z(i), j)$ were defined by Voevodsky and represent motivic cohomology (higher Chow groups). When working over the field $\mathbb{R}$ of real numbers, the assignment sending a smooth variety X to its set of real points $X(\mathbb{R})$ with the euclidean topology extends to the "real realization" functor from motivic spaces (over $\mathbb{R}$) to ordinary spaces. In this talk I will explain what the real realization of the $K(Z(i), j)$ is. Perhaps slightly surprisingly, the answer is somewhat more complicated than for complex realization (which sends a motivic Eilenberg-MacLane space to an ordinary Eilenberg-MacLane space).

Alexander Betts: *Extensions in the fundamental group and ℓ-adic Ceresa classes*

The Ceresa cycle is an algebraic cycle attached to any pointed curve (X, x), whose attached cohomology classes control the extension between the first and second graded pieces of the fundamental group of (X, x). A classic question is to study for which curves (X, x) this extension in the ℓ-adic fundamental group splits: for instance, when X is hyperelliptic and x is Weierstrass, this extension is known to split, but it is known not to split for the generic curve of genus $g \geq 3$. In this talk, I will describe ongoing joint work with Wanlin Li, in which we study this question when the base field is p-adic. When p is different from ℓ, the extension always splits for weight reasons, but when p is equal to ℓ, we show that the extension is actually non-split for "most" pointed curves (X, x). The methods we use are primarily p-adic Hodge-theoretic in nature.

Anna Cadoret: *On the trivial locus of p-adic local systems*

Let k be a number field, X a smooth, geometrically connected variety over k and V a p-adic local system on X. The trivial locus of V is the set of closed points x of X where x^*V

has finite monodromy. The following is a consequence of the unramified Fontaine-Mazur conjecture:
Conjecture: The trivial locus of V is empty unless V itself has finite monodromy.
I will discuss evidences towards this conjecture; in particular explain why it holds if V is part of a $\mathbb{Q}$-compatible family (e.g. $V = \mathbb{R}^2$ if $f_*\mathbb{Q}_p(i)$ for $f : Y \to X$ a smooth proper morphism). This is a joint work with Akio Tamagawa.

Thomas Geisser: *Brauer and Neron-Severi groups of surfaces over finite fields.*

For a smooth and proper surface over a finite field, the formula of Artin and Tate relates the behavior of the zeta-function at 1 to other invariants of the surface. We give a refinement which equates invariants only depending on the Brauer group to invariants only depending on the Neron-Severi group, and give estimates of the terms appearing in the formula. If time permits we discuss applications to abelian varieties and K3-surfaces.

Peter Haine: *Reconstructing schemes from their étale topoi*

In Grothendieck's 1983 letter to Faltings that initiated the study of anabelian geometry, he conjectured that a large class of schemes can be reconstructed from their étale topoi. In this talk, I'll discuss joint work with Magnus Carlson and Sebastian Wolf that proves Grothendieck's conjecture for infinite fields. Specifically, we show that over a finitely generated field k of characteristic 0, seminormal finite type k-schemes can be reconstructed from their étale topoi. Over a finitely generated field k of positive characteristic and transcendence degree ≥ 1, we show that perfections of finite type k-schemes can be reconstructed from their étale topoi. Our results generalize work of Voevodsky.

Tim Holzschuh: *On the generalised real Section Conjecture*

We sketch a proof of the generalised pro-2 real Section Conjecture for what we call equivariantly triangulable varieties over $\mathbb{R}$. Examples include all smooth varieties as well as all (possibly singular) affine/projective varieties. Building on this, we derive the generalised real Section Conjecture in the geometrically étale simply connected case.

Marc Hoyois: *Motivic spectra without $\mathbb{A}^1$-invariance*

I will talk about recent results in the theory of motivic spectra without $\mathbb{A}^1$-invariance, in particular Atiyah duality and applications to logarithmic cohomology theories. This is joint work with Toni Annala and Ryomei Iwasa.

Katharina Hübner: *Tame fundamental groups I. Rigid spaces*

This is the first of two talks on a joint project of P. Achinger, K. Hübner, M. Lara, and J. Stix.

p-adic analytic objects such as the affinoid unit disc often have complicated fundamental groups. In order to deal with this difficulty, we introduce the notion of a tamely ramified finite etale cover of a rigid analytic space, defined in terms of its ramification at the points of Huber's "universal compactification". The resulting "tame fundamental group" is expected to have good finiteness properties. Our main result is that it is (topologically) finitely generated (for a quasi-compact and quasi-separated rigid space over an algebraically closed field).

In this talk, I will define the tame fundamental group and present the first part of the proof of its finite generation. At the end, the problem will be reduced to an analogous question concerning certain (non-fs) log schemes over the residue field, arising as special fibers of semistable formal models. This will be discussed in the subsequent talk of P. Achinger.

Moritz Kerz: *A non-abelian version of Deligne's fix part theorem*

A fixed part theorem provides a criterion for extending cohomological data from a fiber to the total space of a fibration. In collaboration with H. Esnault, we revisit and expand upon earlier work of Jost-Zuo and Katzarkov-Pantev on the fixed part theorem for variations of Hodge structures. The key new aspect is the systematic use of the algebraic monodromy group of the underlying local system.

Daniel Litt: *Integrality and algebraicity of solutions to differential equations*

Eisenstein proved, in 1852, that if a function $f(z)$ is algebraic, then its Taylor expansion at a point has coefficients lying in some finitely-generated $\mathbb{Z}$-algebra. I will explain ongoing joint work with Josh Lam which studies the extent to which the converse of this theorem holds. Namely, we conjecture that if $f(z)$ satisfies a (possibly non-linear!) algebraic ODE, non-singular at 0, and its Taylor expansion has coefficients lying in a finitely-generated $\mathbb{Z}$-algebra, then f is algebraic. For linear ODE, we prove this conjecture when (A) $f(z)$ satisfies a Picard-Fuchs equation, with initial conditions the class of an algebraic cycle, and in some other cases. For non-linear ODE, we prove it when $f(z)$ satisfies an "isomonodromy" ODE with "Picard-Fuchs" initial conditions.

Catrin Mair: *Condensed Shape of a Scheme*

The ∞-category $Cond(Ani)$ of condensed anima combines homotopy theory with the topological space direction of condensed sets. For example, we can recover the shape of a sufficiently nice topological space from the corresponding condensed anima. My talk will focus on a joint refinement of the étale homotopy type and the pro-étale fundamental group of a scheme, realised as an object in $Cond(Ani)$. This condensed version of a homotopy type, which I will refer to as condensed shape, is closely related to the work of Barwick, Glasman and Haine in the *Exodromy* paper.

Alberto Merici: *Tame and logarithmic motivic homotopy*

It is well known that integral p-adic étale cohomology theories do not fit well with motivic homotopy theory because of the Artin—Schreier sequence. In recent years, two approaches to this problem emerged: motives for log schemes of Binda-Park-Østvær and the tame cohomology of Hübner-Schmidt. In this talk, I will prove a comparison theorem between the two theories, which implies some motivic properties of tame cohomology, as conjectured by Hübner-Schmidt.

Fabien Morel: *On the $\mathbb{A}^1$-fundamental group and cellular $\mathbb{A}^1$-homology of smooth $\mathbb{A}^1$-connected k-schemes*

After some slight recollections, I will explain the notion of smooth $\mathbb{A}^1$-connected k-schemes (k a field), and give some examples and computations of the $\mathbb{A}^1$-fundamental group of those.

I will then focus on surfaces and, based on some joint work with Aravind Asok, I will discuss the classification of projective smooth $\mathbb{A}^1$-connected surfaces when k is algebraically closed. This involves very much the $\mathbb{A}^1$-fundamental group, as in topology. In the last part, based on a joint work with Anand Sawant, I will define the $\mathbb{A}^1$-cellular homology of smooth schemes, and give some examples of computations and explain why it is a more geometric version of the $\mathbb{A}^1$-homology. I will explain also some conjectures relating the top $\mathbb{A}^1$-cellular homology of a projective smooth k-scheme and its orientability, and explain why for rational projective smooth k-surfaces, this is known.

Timo Richarz: *Categorical Künneth formulas*

In my talk, I will discuss ongoing joint work with Jakob Scholbach focused on developing Künneth formulas for motives within a categorical framework.

Oliver Röndigs: *Filtering Hermitian K-theory over number rings*

Recent work of Calmes, Harpaz, and Nardin, based on their collaboration with Dotto, Hebestreit, Land, Moi, Nikolaus, and Steimle, provides a reasonable motivic spectrum with good properties representing Hermitian K-theory in the motivic stable homotopy category of S. Here S can be any regular Noetherian base scheme of finite Krull dimension; the inconvenient restriction that 2 be invertible on S is not required anymore. The talk will discuss slice filtrations on that motivic spectrum, which have previously been studied under this inconvenient restriction. If time permits, the slice filtrations will be used in the special case to compute Hermitian K-theory of number rings. This is joint work with Haakon Kolderup and Paul Arne Østvær.

Tamas Szamuely: *The generalized height pairing over complex function fields*

With D. Rössler we have defined a generalization of Beilinson's geometric height pairing for algebraic cycles on smooth projective varieties defined over the function field of a variety B. Our height pairing has values in the second cohomology of B but conjecturally should come from a geometric pairing with values in the $\mathbb{Q}$-Picard group. Using techniques of mixed Hodge modules, we can produce such a pairing unconditionally in the case when the base field is $\mathbb{C}$. The construction is in fact of motivic nature. This is joint work in progress with P. Brosnan and G. Pearlstein.

Kirsten Wickelgren: *On quadratically enriched logarithmic zeta functions*

We enrich the logarithmic derivative of the zeta function to a power series with coefficients in the Grothendieck-Witt group of stable isomorphism classes of unimodular bilinear forms, using traces of powers of Frobenius in A1-homotopy theory. Building off of work of Morel-Sawant and Bondarko, we construct a symmetric monoidal chain functor from smooth schemes to bounded complexes of homotopy modules. We show the quadratically enriched logarithmic zeta function to be connected to the Betti numbers of the associated real manifold under various restrictions. This is joint work with Margaret Bilu, Wei Ho, Padma Srinivasan, Isabel Vogt, and joint work with Tom Bachmann.