COMPLEX MULTIPLICATION CYCLES ON GSPIN SHIMURA VARIETIES

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In this Junior seminar, we want to understand the degree of complex multiplication cycles on the integral model of Gspin Shimura varities. It plays an important role in a conjecture of Bruinier and Yang, relating the arithmetic intersection multiplicities of special divisors and complex multiplication points on Shimura varieties to the central derivatives of certain L-functions.

The seminar is organized as follows. After recalling the definition of Gspin Shimura varieties M associated to the group of spinor similitudes of a quadratic space V over $\mathbb{Q}$ of signature $(n, 2)$, we will construct its integral model $\mathcal{M}$ via Kuga-Satake abelian schemes. Then we show the moduli stack of elliptic curves with complex multiplication can be realized as cycles of $\mathcal{M}$ associated to subspaces of V with signature $(0, 2)$. We end by calculating the degrees of complex multipication cycles. The main reference is [1, §2 & 4].

The seminar will take place on Monday 17.06.2024 in room 51 (on the ground floor in S2 15) at Darmstadt. We will go to have lunch after talk 2, and have the coffee break after talk 3.

Talk 1. Gspin Shimura varieties (9:15-10:15)

This talk should introduce the Shimura variety M defined by a reductive group over $\mathbb{Q}$ of type $\mathrm{GSpin}(n, 2)$ [1, §2.2]. Then define the Hodge structures on $M(\mathbb{C})$ and construct the associated Kuga-Satake abelian scheme, see also [4, §3].

Talk 2. The integral models (10:30-11:30)

This talk should construct the integral model $\mathcal{M}$ of the Shimura variety M [1, §2.4]. The idea is to construct the integral model over $\mathbb{Z}_{(p)}$ (for $p \neq 2$, see [3] for more details) as the normalization of Siegel moduli stack in $M^{\diamond}$ firstly, then glue them to an integral model over $\mathbb{Z}$ [1, Proposition 2.4.3, 2.4.6].

Lunch (11:45-13:15)

Talk 3. Special endmorphisms and special divisors (13:30-14:45)

This talk should introduce the special endmorphisms and use them to define the special divisors [1, §2.6 - 2.7], see also [4, §5]. Then prove that the special divisor is a Cartier divisor on $\mathcal{M}$ [1, Proposition 2.7.4].

Coffee Break (14:45-15:30)

Talk 4. Complex moultipication cycles (15:30-16:30)

Recall the basic properties of Clifford algebras and the zero dimensional Shimura varieties Y associated to quadratic spaces of signature $(0, 2)$ [1, §4.2-4.3]. Then show that the integral model $\mathcal{Y}$ of Y can be realized as the moduli stack of elliptic curves with complex multiplication by $\mathcal{O}_k$ [1, §4.3], see also [4, §6].

Talk 5. Degree of special divisors (16:45-17:45)

This talk should define the special endmorphisms and the special divisors $\mathcal{Z}_0(m, \mu)$ on the arithmetic curve $\mathcal{Y}$, and prove Theorem 4.5.1 concerning the degrees of $\mathcal{Z}_0(m, \mu)$ [1, §4.4-4.5].

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References

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[3] M. Kisin, Integral models for Shimura varieties of abelian type, J. Amer. Math. Soc. 23 (2010), 967-1012.

[4] K. Madapusi Pera, Integral canonical models for spin Shimura varieties, Compositio Math. 152 (2016), 769-824.