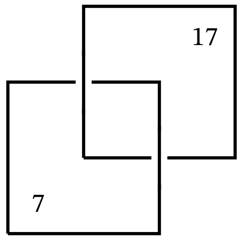
GAUS JUNIOR AG: KNOTS AND PRIMES



linking number $\mathrm{lk}_2(7,17) = 1$

Arithmetic topology is about similarities between the ring of integers of a number field and 3-manifolds. Some similarities you should keep in mind:

Sphere $X = S^3$	**Integers** $X = \mathrm{Spec}(\mathbb{Z})$
Knot $k : S^1 \hookrightarrow X$	**Finite prime** $p : \mathrm{Spec}(\mathbb{F}_p) \hookrightarrow X$
Tubular neighbourhood V_k of k	p**-adic integers** $V_p = \mathrm{Spec}(\mathbb{Z}_p)$
Boundary torus ∂V_k	p**-adic numbers** $\mathrm{Spec}(\mathbb{Q}_p)$
$1 \to \mathrm{I}_k \to \pi_1(\partial V_k) \to \pi_1(V_k) \to 1$, the **meridian** α generates I_k, the **longitude** β generates $\pi_1(V_k)$ and relation $[\alpha, \beta] = 1$	$1 \to \mathrm{I}_p \to \pi_1(\mathrm{Spec}(F_p)) \to \pi_1(V_p) \to 1$, the **monodromy** τ generates $\mathrm{I}_p^{\mathrm{tame}}$, the **Frobenius** frob_p generates $\pi_1(V_p)$ and relation $[\tau, \mathrm{frob}_p] = \tau^{1-p}$
Knot complement $X_k = X \setminus k$	**Prime complement** $X_p = X \setminus p$
Knot group $\pi_1(X_k)$	**Prime group** $\pi_1(X_p)$
Cyclic cover $h_\infty : X_k^\infty \to X_k$ and $\mathrm{Gal}(X_k^\infty / X_k) \simeq \mathbb{Z}$	**Pro-**p**-cyclic cover** $h_\infty : X_p^\infty \to X_p$ and $\mathrm{Gal}(X_p^\infty / X_p) \simeq \mathbb{Z}_p$

Talk 1 (Thorger). $\pi_1(S^1), \pi_1(T^2), \pi_1(S^3)$ [1, example 2.1, 2.2, 2.3]. Presentation of the knot group [1, example 2.6]. Structure of the link group [1, example 7]. Cyclic covers of a knot complement [1, example 2.12]. Fox completion [1, example 2.14] . Ramification of knots [1, chapter 5.1].

Linking numbers [1, proposition 4.1]. If time permits, combinatorical nature of the Alexander polynomial using Skein relations and example [2].

Talk 2 (Leonie). Unramified extensions of number fields [1, page 34]. $\pi_1(\kappa(p)), \pi_1^t(F_p)$ [1, example 2.25, 2.34, 2.39]. Cyclotomic extensions of $\mathrm{Spec}(\mathbb{Z}) \setminus S$ [1, example 2.46]. Ramification of primes [1, chapter 5.2].

Artin-Verdier site and its sheaves [3, definition 2.1, proposition 2.3]. Artin-Verdier duality [3, theorem 2.12], [4, theorem 5.4]. Cohomology of the ring of integers [3, corollary 2.15], [4, proposition 2.9].

Legendre symbols as an analogue of modulo 2 linking numbers [1, proposition 4.4].

Talk 3 (Tim)**.** Differential modules and their representation matrices [1, definition 9.1, theorem 9.5, corollary 9.6]. Alexander module, matrix, ideal, and polynomial [1, example 9.7].

Alexander polynomial and homology groups [1, theorem 9.8, equation 11.1]. Asymptotic formula [1, theorem 11.4].

Talk 4 (Jaro)**.** Complete differential modules and their representation matrices [1, definition 9.10, theorem 9.14, corollary 9.15]. Complete Alexander module, ideal [1, example 9.16]. Iwasawa module, arithmetic Alexander polynomial, and Iwasawa polynomial [1, theorem 9.17, example 9.18], [5].

Iwasawa polynomial and class groups [1, equation 11.3]. Iwasawa's class number formula [1, theorem 11.11].

Talk 5 (Alireza)**.** Sato-Tate in arithmetic and topology.

References

[1] Masanori Morishita. Knots and primes: An introduction to arithmetic topology, 2011.

[2] Vaughan F. R. Jones. The jones polynomial, 2005. URL `https://math.berkeley.edu/~vfr/jones.pdf`.

[3] Eric Ahlqvist and Magnus Carlson. The étale cohomology ring of the ring of integers of a number field, 2018. URL `https://arxiv.org/abs/1803.08437`.

[4] Mel Bienefeld. An étale cohomology duality theorem for number fields with a real embedding, 1987. URL `https://www.ams.org/journals/tran/1987-303-01/S0002-9947-1987-0896008-0/S0002-9947-1987-0896008-0.pdf`.

[5] Jürgen Neukirch, Alexander Schmidt, and Kay Wingberg. Cohomology of number fields, 2000. URL `https://www.mathi.uni-heidelberg.de/~schmidt/NSW2e/`.