GAUS AG on Moduli of Langlands parameters
Winter term 22/23

Program by Thibaud van den Hove, Judith Ludwig

Let G be a reductive group over a non-archimedean (p-adic) local field F. Langlands parameters form one side of the local Langlands correspondence for G. In recent years, the philosophy has emerged that this correspondence should not just be a map of sets satisfying certain properties, but instead a categorical equivalence, cf. [Zhu20], [FS21], [BZCHN20], [Hel20a], and many other references. Very roughly, on the automorphic side, the smooth irreducible $G(F)$-representations should be replaced by sheaves on the stack Bun_G classifying G-bundles on the Fargues-Fontaine curve (which encorporates the representation theory of all pure inner forms of G), while on the spectral side, Langlands parameters should be replaced by sheaves on their moduli space.

The main goal of the seminar is to study the geometry of the moduli space of Langlands parameters, following [DHKM20]. The last talks address representation-theoretic applications as in [DHKM22].

The seminar takes place weekly, Tuesdays 14:00-15:30, in hybrid format via Zoom (Meeting-ID: 612 2072 7363; Password: Largest six digit prime). The first meeting is on October 18; the last meeting on February 7 is a double session with dinner afterwards taking place in Darmstadt or Heidelberg.

Talk 1 (**L-parameters. 18.10, Alireza Shavali**). Recall the definition of the Weil group of a non-archimedean local field. Give some recollections about reductive groups, in order to define the L-group and L-homomorphisms. Give the different possible definitions [DHKM20, (1)-(4) on pages 2 and 4] of L-parameters. Explain why these agree for ℓ-adic coefficients, using Grothendieck's ℓ-adic monodromy theorem, and why they differ for more general coefficients.

Talk 2 (**The space of tame parameters. 25.10, Sriram Chinthalagiri Venkata**). Explain how one can view L-parameters as cocycles, define the moduli functor $\underline{Z}^1(W_F^0, \hat{G})$, and show it is representable by a scheme [DHKM20, p5]. Specializing to the tame case, construct the scheme $\underline{Z}^1(W_F^0/P_F, \hat{G})$ [DHKM20, §2.1] and study its geometry [DHKM20, §2.2].

Talk 3 (**Geometry and the universal family in the tame case. 8.11, Timo Richarz**). Continue the study of the geometry of $\underline{Z}^1(W_F^0/P_F, \hat{G})$ [DHKM20, §2.3]. Explain how to define ℓ-adic continuity for more general coefficient rings, and construct the universal ℓ-adically continuous L-parameter [DHKM20, §2.4]. Time permitting, explain how instead of [DHKM20, Definition 2.11], one could deal with continuity for general coefficients using condensed mathematics as in [FS21, §VIII].

Talk 4 (**Moduli spaces of cocycles and their quotients. 15.11, Judith Ludwig**). The goal of this talk is to describe moduli spaces of cocycles and their quotients by conjugation actions, which will later be applied to L-parameters. More precisely, cover [DHKM20, §A.1,§A.2]. Then recall some background on GIT quotients, and explain the relation between the GIT quotient and the sheaf quotient of moduli of cocycles [DHKM20, §A.3].

Talk 5 (**Representatives of L-parameters. 22.11, Jakob Burgi**). Continue the study of moduli of cocycles [DHKM20, §A.4, §A.5, §A.6], and deduce [DHKM20, Theorem 3.1].

Talk 6 (**Reduction to tame parameters. 29.11, Manuel Hoff**). Prove [DHKM20, Theorem 3.4]. State [DHKM20, Theorem 3.12] and, time permitting, explain how one can prove it by modifying the proof of [DHKM20, Theorem 3.4], cf. [DHKM20, Remark 3.9].

Talk 7 (**Geometry of the moduli space. 6.12, Can Yaylali**). Cover the first half of [DHKM20, §4], up to Theorem 4.13 and its proof.

Talk 8 (**Connected components of the moduli space. 13.12, Patrick Bieker**). Finish [DHKM20, §4]. The most important results are [DHKM20, Proposition 4.17, Theorem 4.18, Proposition 4.23, Theorem 4.29, Corollary 4.30].

Talk 9 (**Unobstructed points. 20.12, Rızacan Çiloğlu**). Cover [DHKM20, §5.1, §5.2], ending with a proof of [DHKM20, Theorem 5.5]. (In the course of the proof, it is not necessary to go through the classification of reductive groups. Instead, it is enough to illustrate what happens in general by covering the simpler cases.)

Talk 10 (**Description of the GIT quotient. 10.1, Gebhard Böckle**). Introduce the notion of banal primes [DHKM20, §5.3]. Describe the GIT quotient of the moduli of L-parameters by the conjugation action, first integrally [DHKM20, §6.1], and then over algebraically closed fields of banal characteristic [DHKM20, §6.2].

Talk 11 (**Link to Galois deformation theory. 17.1, Alireza Shavali**). Explain some background on Galois deformation theory (cf. [Gee22, §2,§3], [Böc13, §1, §3.2]) focusing on introducing the universal framed deformation ring $R^{\square}_{\overline{\rho}}$ of a local Galois representation $\overline{\rho} : G_F \to \mathrm{GL}_n(\overline{\mathbb{F}}_\ell)$. For $G = \mathrm{GL}_n$, show that the completion of the local ring of the moduli space of L-parameters at an $\overline{\mathbb{F}}_\ell$-point $x_{\overline{\rho}}$ recovers $R^{\square}_{\overline{\rho}}$ (cf. [Hel20b], [Zhu20, p.20-21]).

Talk 12 (**Finiteness of L-parameters. 24.1, Torsten Wedhorn**). This talk follows [DHKM22, §2]. Cover the proof of [DHKM22, Theorem 2.3], and deduce [DHKM22, Corollaries 2.4 and 2.5].

Talk 13 (**Finiteness of Hecke algebras. 7.2, Jean-François Dat**). Give an overview of excursion operators and excursion algebras, as in [FS21]. Use this to prove [DHKM22, Theorems 1.1 and 1.2], following [DHKM22, §3].

Talk 14 (**Second adjointness. 7.2, Jean-François Dat**). Prove [DHKM22, Corollary 1.3], following [DHKM22, §4]. Discuss a number of consequences, such as [DHKM22, Corollaries 1.4, 1.5 and 1.6], depending on the available time.

References

[Böc13] Gebhard Böckle. Deformations of Galois representations. In *Elliptic curves, Hilbert modular forms and Galois deformations.*, pages 21–115. Basel: Birkhäuser/Springer, 2013.

[BZCHN20] David Ben-Zvi, Harrison Chen, David Helm, and David Nadler. Coherent Springer theory and the categorical Deligne-Langlands correspondence. *arXiv preprint arXiv:2010.02321*, 2020.

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[Hel20a] Eugen Hellmann. On the derived category of the Iwahori-Hecke algebra. *arXiv preprint arXiv:2006.03013*, 2020.

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