- Example of connected scheme locally of finite type that is not of finite type

- $\mathrm{Spec}(\mathbb{C}) \to \mathrm{Spec}(\mathbb{R})$
  $\mathrm{Spec}(\mathbb{R}) \to \mathrm{Spec}(\mathbb{Q})$ of finite type?

  More general: $K \to L$ field extension

  When is $\mathrm{Spec}(L) \to \mathrm{Spec}(K)$ of finite type?

- Why $\mathbb{P}^n_R$ is separated? Hint: $\mathbb{P}^n_R = U \cup V$
  $U, V \cong \mathbb{A}$?

- $X \xrightarrow{f} S$ map of schemes

  (1) $U \overset{j}{\hookrightarrow} X$ subscheme, f separated

  $\Rightarrow f \circ j = f|_U$ separated?

  (2) Suppose $\exists (U_i)_{i \in I}$ open covering of X

  s.t. $f|_{U_i} : U_i \to S$ separated

  $\overset{?}{\Rightarrow} f$ separated

- $U \overset{j}{\hookrightarrow} X$ open immersion

$\overset{?}{\Longrightarrow}$
- j of finite type (!!)
- j separated
- j proper

- R principal ideal domain, $p \in R$ prime elt.

$\Longrightarrow R_{(p)}$ discrete valuation ring

- R ring, $n \geq 0$.

Show that $\mathbb{P}^n_R \longrightarrow \operatorname{Spec}(R)$ is proper

Hint: Use $\mathbb{P}^n_R \cong \mathbb{P}^n_{\mathbb{Z}} \times_{\operatorname{Spec}(\mathbb{Z})} \operatorname{Spec}(R)$

$\Rightarrow$ W.l.o.g.: $R = \mathbb{Z}$, in particular noetherian

Use valuative criterion, use that for

A local (e.g. A dvr or field)

$\mathbb{P}^n_{\mathbb{Z}}(A) = \{ [x_0, \dots, x_n] \in A^{n+1}/\sim \; ; \text{ some } x_i \in A^\times \}$

$[x_0, \dots, x_n] = [y_0, \dots, y_n] :\Longleftrightarrow \exists \alpha \in A^\times : y_i = \alpha x_i \; \forall i$